Vertical Mergers with Bilateral Contracting and Upstream and Downstream Investment

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Abstract

We extend the theory of bilateral vertical contracting to a double moral hazard setting where upstream and downstream firms make complementary investments that enhance demand; downstream firms make fixed investments to enter the downstream market, and contracts are private information and determined through simultaneous bilateral bargaining. We show that vertical mergers mitigate bilateral contracting externalities and hold-up, which leads to an increase in complementary investments. If downstream products are either sufficiently distant or sufficiently close substitutes, a vertical merger benefits the merging firm, consumers, *and* the unintegrated downstream firm. For intermediate degrees of product differentiation, a case with linear demand and quadratic investment costs shows that consumers benefit if the marginal cost of investment is sufficiently low as revealed, for example, by a high ratio of R&D investments to sales. We apply the model to a vertical merger in the computer industry.

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I. Introduction

A. Overview

In a leading theory of harm from vertical mergers, bilateral contracting between upstream and downstream firms leads to marginal prices for the inputs sold by the upstream firm that are too low to maximize the joint profits generated by the upstream firm's products.[1] Notably, the source of the low input prices is an externality problem: When negotiating bilaterally with a given downstream firm, the upstream firm (jointly with the negotiation partner) treats sales stolen from downstream rivals as increasing profits, even though they do not increase total profits for the "ecosystem" that sits under the upstream firm. As such, the series of bilateral negotiations fails to maximize the profits to be shared between the upstream firm that sits at the top of the ecosystem and its downstream partners. In simple terms, the externalities due to bilateral bargaining prevent the upstream firm and its partners from fully capturing the additional area under the demand curve created by their investments. While failure to internalize this externality leads to lower input prices, which is generally viewed as a good thing in antitrust analysis, the failure to internalize the externality—and thus the failure to maximize profits flowing from the upstream firm's input—reduces investment incentives and thus correspondingly reduces total and consumer welfare.

The effect of vertical mergers in bilateral contracting models flows from the fact that the parties to a vertical merger internalize the bilateral contracting externality. One effect of this is to put upward pressure on wholesale prices charged to unintegrated downstream firms, which creates a source of harm to competition from vertical mergers. And because wholesale prices can be low in the absence of the merger, double marginalization benefits can be small in, meaning that, when considering prices alone, a vertical merger can be anticompetitive in such bilateral contracting models.[2]

In this paper, we show that once investment effects are incorporated into such models, the negative effects on welfare can be reversed. In particular, we show that when the investment

[1] This contrasts with situation of public take-it-or-leave-it contract offers, where an upstream monopolist can induce the fully-integrated outcome with nonlinear contracts. See, e.g., Mathewson & Winter (1984).

[2] An early treatment of bilateral contracting externalities with linear input contracts is O'Brien (1988), which shows that under simultaneous Nash bargaining by an upstream monopolist and downstream Cournot oligopolists, wholesale prices will be below the level that induces the vertically integrated outcome if the downstream market is sufficiently competitive. Under linear demand, constant marginal cost, and downstream Cournot competition, a vertical merger between the upstream firm and one downstream firm raises the final product price if there are more than two downstream firms (O'Brien, 2008). See also Sheu and Tarragin (2021). With nonlinear input contracts, vertical mergers (or vertical restraints that restore the integrated outcome) can raise price in a broader class of cases. See Hart & Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994), Rey and Verge (2005, 2020), Rey and Tirole (2007).

benefits from internalizing the externality—together with the investment benefits from eliminating "hold-up" between the integrating firms—are incorporated, vertical mergers are pro-competitive in a wide range of settings.[3]

Our result formalizes intuition from existing economic literature that bilateral contracting theories are flawed because they miss effects on investment incentives. For example, Carlton and Heyer (2007) draw a distinction between upstream firms' strategies that involve extraction—i.e., capturing the value of their investments—and those that involve the extension of market power into other markets. A principle they advance is that the main focus of antitrust laws toward single firm conduct should be on extension rather than extraction because extraction is necessary to call forth investment.[4] Our result formalizes that principle.

Even if one takes the view that there can be legitimate antitrust claims against strategies that involve extraction, antitrust policy toward strategies that have a clear extraction element should certainly still incorporate the effects of the policy on investment incentives. For example, vertical mergers combine complementary assets, which means that direct effects of the merger can involve both: (i) internalizing externalities from the independent pricing of complements, and (ii) internalizing externalities from independent decisions regarding complementary investments.

Although antitrust inquiries into vertical and conglomerate complements mergers historically have focused on price effects, investment effects also clearly matter, in some cases more than price effects. In the computer industry, for example, producers of computer chips (CPUs, GPUs, etc.) and software make large investments to produce complementary products required to run various types of computers. Outside of the extreme case where independent producers can write complete contracts over all relevant aspects of their complementary investment decisions (thus fully dealing with hold-up and moral hazard issues), standard theory predicts that they are likely to invest less when operating independently than when merged.[5] Upward pressure on investment from complementary mergers is the precise analog of the downward pricing pressure from eliminating double marginalization or the Cournot complements problem. And when investments are important for value creation and contracts are incomplete, upward pressure on investment from a vertical (or,

[3]The "hold-up" problem in this context refers to a situation where a producer that makes up-front investments is "held-up" by complementary producers that bargain for a share of the gains from investments they did not make. See Goldberg (1976), Klein, Crawford, and Alchian (1978), Williamson (1979), and Tirole (1988), Section 1.3. Klein, Crawford & Alchian and Williamson use the terms "appropriation" and "opportunism" instead of "hold-up" to mean essentially the same thing.

[4]This issue is also discussed in Cooper et al. (2005) and Rey and Tirole (2007).

[5]This is true when the merged firm can specify, monitor, and enforce investment efforts that are not fully contractible when the firms are independent.

more generally, complements) merger is as fundamental as potential upward pressure on price from a horizontal merger, and thus it cannot be ignored in assessing the merger's all-in effects on competition and welfare.

While many discussions of investment effects from vertical mergers focus on the upstream firm, effects on the investment incentives of downstream firms can be equally relevant. In bilateral contracting settings where firms divide the gains from trade, any dissipation of joint profits due to bilateral contracting externalities also dissipates the incremental profits from downstream investment.

Further, upstream and downstream investment incentives typically interact. For example, upstream sellers often benefit from having many differentiated downstream sellers making investments, and the incentive for downstream firms to enter and invest depends on the extent of upstream investment, which can be quite important for the overall quality of the ecosytsem. Bilateral contracting externalities that dissipate joint profits may reduce both upstream and downstream incentives to invest, thereby hampering the development of the vertical chain of production and relationships that work together to deliver end products to consumers.

B. Summary of Model and Findings

Our model extends the standard bilateral contracting model of vertical mergers to the case where upstream and downstream firms make investments that enhance demand prior to bargaining over contracts. An upstream supplier with market power potentially sells a product through two competing downstream sellers. The upstream firm and one of the downstream firms also decide whether to merge. Remaining downstream firms then decide whether to make a fixed investment necessary to enter the downstream market (which might be interpreted as introducing a new product),[6] and the upstream firm and downstream firms that enter make additional investments that shift the final demand for the product at a convex cost. After investments are made, the upstream and downstream firms bilaterally negotiate (via simultaneous Nash bargaining) private nonlinear contracts. Downstream firms then engage in Bertrand competition in the downstream market.

We analyze the effects of vertical mergers in this setting. We first show that when investments are complementary, the investments of unintegrated firms are lower than the investments a fully

[6] An important factor in the datacenter market application discussed in Section V is that chipmakers (downstream firms) have tried to enter using Arm's IP (the upstream product) but have failed because the Arm ecosystem is under-developed. The downstream entry decision in our model allows us to capture this effect. Potential entrants that understand the importance of investment in the ecosystem will not enter if they anticipate that there will be too little investment in the ecosystem due to the externalities associated with bilateral bargaining, and the promotion of such entry is another potentially important benefit of a vertical merger that mitigates these externalities.

integrated firm would make.[7] Underinvestment by unintegrated firms occurs for two reasons.

- First, as described above, bilateral contracting externalities lead to downstream prices below the joint profit maximizing levels, which means that unintegrated firms have smaller combined incremental benefits from investment than the incremental benefits from investment that accrue to a fully integrated firm. This leads unintegrated firms to invest less than a fully integrated firm.

- Second, in the absence of vertical integration, ex post bargaining between the upstream firm and each downstream firm creates a hold-up situation where some of the incremental value generated by each firm's investment is captured by other firms, thus reducing ex-ante investment incentives. A fully integrated firm does not experience hold-up and therefore captures the full value of each investment, leading to higher investment than chosen by unintegrated firms.

We stress that the effect of bilateral contracting externalities on investment and the effect of bargaining on investment are distinct phenomena, each of which our model captures. When bilateral contracting externalities are present, underinvestment would still occur even if each investing firm captured all available incremental profit from the investment (that is, even in the absence of hold-up). This is true because that the incremental profit from investment available to the unintegrated firms is less than the incremental profit available to a fully integrated firm (that does not face bilateral contracting externalities), reducing investment incentives. Similarly, hold-up due to bargaining exists in our model even if one firm has all the bargaining power. This is true because investment in the model involves double moral hazard, where both the upstream firm and each downstream firm make demand-enhancing investments. Although greater bargaining power for the upstream firm (say) means that it faces a smaller hold up problem, it also means that downstream firms face a bigger hold-up problem, and vice versa. Thus, hold-up in our model is not an artifact of the distribution of bargaining power, but rather is an intrinsic feature of environments with ex ante investment and double moral hazard.[8]

[7] Although full integration typically is not the relevant benchmark in a merger case, a comparison of the no integration outcome with the fully integrated outcome in our model illuminates the economic forces that determine the effects of a vertical merger. It also facilitates comparison with the existing literature on vertical contracting, which often compares the outcome that can be achieved through contracting with the fully integrated outcome.

[8] An alternative timing assumption in our model would have bargaining take place prior to investments, so that there is no hold-up due to bargaining, and so that contracts can be structured to induce investment and pricing behavior that maximizes bilateral profits, as in typical moral hazard models. However, even there, because our model involves double moral hazard, pre-integration investment would still be suboptimal with this alternative timing unless

Our model embeds both bilateral contracting externalities and hold-up due to bargaining in a model with upstream and downstream investments and downstream pricing, allowing us to weigh the conflicting forces of a vertical merger on prices and investments. The main result of the paper is that when upstream and downstream investments are strategic complements—a natural assumption in settings with complementary investments—a vertical merger benefits the merging firms and consumers if downstream products are sufficiently distant or sufficiently close substitutes. A corollary is that a vertical merger also benefits the unintegrated downstream firm if products are sufficiently distant or close substitutes. Notably, however, the merger may benefit consumers in many cases where it harms unintegrated rivals, meaning that rival's objections to the merger are not a valid screen of the competitive effects of the merger, an important lesson for antitrust decision makers.

Intuition for our main result is as follows. If downstream products are distant substitutes, then: (i) the pre-merger dissipation of joint profits due to bilateral contracting externalities is small, and (ii) the merger has small price effects, but the hold-up problems from pre-merger bilateral bargaining are magnified, and the merger raises investment by eliminating hold-up.[9] Consumers benefit because the increase in consumer surplus caused by greater investment outweighs the negative effect on consumer surplus from higher prices. If, on the other hand, downstream firms are close substitutes, hold-up problems are smaller, but the dissipation of joint profits due to bilateral contracting when both firms enter is large. If downstream margins are low enough that only one firm enters prior to the merger, the merger raises investment by eliminating hold-up,[10] and it may increase investment enough to make it profitable for the other downstream firm to enter, which increases investment further. Either way, consumers benefit from the merger. In sum, little differentiation (close substitutes) means that the dissipation of joint profits due to bilateral contracting externalities is large, in which case a merger that mitigates these externalties expands investment enough to benefit consumers. And substantial differentiation (distant substitutes) means that joint profits and hold-up due to bargaining is large, in which case a merger that mitigates hold-up expands investment enough to benefit consumers.

firms can specify the details of their investments in the contract (Holmstrom, 1982). Romano (1994) and O'Brien (2017) examine the role of vertical restraints (RPM and all-units discounts) in environments with double moral hazard where contracting occurs in advance of investments. However, their models assume bilateral monopoly. The effects of vertical mergers on investment and pricing in environments with bilateral contracts negotiated in advance of the investments is a topic for further research.

[9]The merger would also raise investment in this case if the timing of investment and contracting were reversed by eliminating double moral hazard.

[10]This would also be true under alternative timing because the merger would eliminate double moral hazard.

For intermediate degrees of product differentiation, the consumer welfare effects of the merger are ambiguous in general. The effects depend on the relative importance of: (i) increased incentives to invest from mitigating hold-up and bilateral contracting externalites vs. (ii) increased incentives to raise prices from mitigating bilateral contracting externalities. To explore the tradeoffs in such cases, we examine a parameterized case with linear demand (a modified Levitan-Shubik (”LS”) demand shifted by investment) and quadratic investment costs. In this case, the merger increases consumer welfare if the marginal cost of investment is below a critical value, which translates into an investment-to-sales ratio that is sufficiently high, making investment a sufficiently important contributor to value.

To illustrate the application of the linear-quadratic model, we apply it to the merger between NVIDIA and Arm in the market for high performance computer chips sold to datacenters. Arm is an upstream licensor of technology used in computer chips designed for datacenters. NVIDIA is a downstream licensee who uses Arm's technology in chip design for chips sold to datacenters. This merger provides a good setting to apply our model because, by all accounts, a major goal of the proposed merger was to expand the use of Arm's technology in datacenters in competition with x86 technology dominated by Intel, an outcome that would require substantial investment by upstream and downstream firms to expand the Arm ecosystem.

The linear-quadratic model predicts that if downstream margins are 60 percent (roughly NVIDIA's gross margin at the time of the proposed (and since abandoned) merger), a vertical merger would increase consumer surplus for all parameter values that yield an investment-to-sales ratio in the industry of greater than about 6 percent. Estimates of the investment-to-sales ratio by high performance chip designers range from 18 to over 50 percent. Thus, the linear-quadratic model we use predicted that the NVIDIA-Arm merger would have benefited consumers. As investments are an important part of the strategies in this industry, an antitrust analysis focused only on price would miss important competitive effects of this merger.

C. Contributions to the Literature and Policy

Our analysis contributes to literatures on bilateral contracting and hold-up and has implications at the intersection of competition policy and intellectual property. The main contribution of the paper is to model explicitly both the price and investment effects of vertical mergers in an environment where firms cannot commit to refrain from negotiating bilaterally—a setting that has been considered relevant in recent regulatory and antitrust proceedings.[11] Notably, our model includes both bilateral contracting externalities and hold-up problems. Our analysis shows that concerns about the investment effects of prohibiting vertical mergers in such environments are valid, and that the net effect of vertical mergers in such environments turns on the degree of product differentiation and the importance of investment for generating incremental value in the competitive process.

An implication of our analysis is that theories of harm from vertical mergers based on bilateral contracting should consider the effects of the merger on investment incentives to determine the net effects on consumers. The logic for doing so is little different than the logic for patent protection: a firm that fails to capture significant benefits from its investments will underinvest. Whereas patent protection prevents the theft of intellectual property from dissipating the value of investments, vertical mergers can prevent bilateral contracting externalities and hold-up from doing the same. Our application of the model to the NVIDIA-Arm proposed merger shows how to use pre-merger investment-to-sales ratios of the merging firms to calibrate a simulation model to predict the competitive effects of vertical mergers in environments with bilateral contracting and ex ante investments.

We are not the first to study how ex ante investments and bilateral contracting interact to determine the effects of vertical mergers. Baake, Kamacke, and Normann (2004) extend Hart and Tirole's model of vertical mergers under private bilateral contracting to the case where the merging upstream firm makes a productive (marginal cost lowering) capacity commitment prior to contracting. In their paper, absent a merger, the upstream firm under-invests in capacity due to profit dissipation caused by bilateral contracting externalities. A vertical merger that mitigates bilateral contracting externalities has two types of effects: higher prices for any given level of investment, and greater investment. Our model differs in several ways: downstream competition is differentiated Bertrand rather than Cournot, ex ante investments are made by downstream firms as well as the upstream firm, and we model the entry decision of one of the downstream firms. We also provide a method for quantifying the welfare effects of vertical mergers in settings with demand-enhancing investment.

In related and independent work, Bisceglia, Padilla, Piccolo and Sheckhar (2022) examine the competitive effects of a vertical merger in a setting where an integrated incumbent with a closed ecosystem competes with an open ecosystem comprised of an upstream supplier (the ecosystem gatekeeper) and two differentiated downstream retailers. In their paper, absent a merger between

[11] See, for example, FCC (2011) and U.S. v. AT&T et. al., 2019.

the gatekeeper and one of the downstream retailers, rent dissipation caused by bilateral contracting externalities leads to low prices for any given level of upstream investment and low ex ante investment by the gatekeeper. A vertical merger mitigates the bilateral contracting externality, which raises prices but also raises the gatekeeper's investment, making the open ecosystem a stronger competitor with the closed ecosystem. Our model differs as follows: we assume a general rather than linear demand structure (except when we need a specific functional form for simulation); we do include competitors who are already integrated; we consider downstream as well as upstream investment, and we endogenize the entry decision of one of the downstream firms.

An interesting contrast to our results comes from the findings of Allain, Chambolle, and Rey (2016), who make a polar opposite assumption about the ability of upstream firms to make commitments. In a model with upstream and downstream duopoly, they assume that upstream firms can commit to be "greedy" in negotiations with downstream firms. Under this assumption, suppliers do not make this commitment prior to the merger, but the merged supplier makes this commitment after the merger and thereby holds up unintegrated downstream firms in a way that harms competition. This finding is reminiscent of that of Ordover, Salop, and Saloner (1990), who showed that a vertically merged firm can benefit from committing to compete less aggressively than it does prior to the merger for the business of unintegrated downstream firms and that such a commitment can harm competition. A distinguishing aspect of our paper (and those of Baake et al. and Bescelia et al.) is to model the effects of vertical mergers when the upstream firm cannot make such commitments. This is in the spirit of the bilateral contracting literature, which assumes that firms cannot credibly commit to refrain from negotiations that they expect to generate positive incremental profits. The difference in results highlights the critical importance of assumptions about the feasible set of commitments when assessing the effects of vertical mergers. If credible commitments to tougher pricing or bargaining are possible, they obviously should be considered. But then one would also need to consider why some commitments are possible but not others, such as commitments by an upstream firm with market power that enable the fully integrated outcome pre-merger (as in Mathewson and Winter, 1984), as such commitments imply no harm from a vertical merger.

Yang (2020) conducts counterfactual analysis based on a structural model of bargaining in the cell phone industry and also finds that vertical mergers can increase investment and generate net benefits for consumers. As in our model, the theoretical model behind Yang's empirical analysis assumes that the upstream and downstream firms make complementary investments and that downstream firms compete in price. Unlike our model, Yang's model assumes that firms negotiate observable linear input prices and that negotiating firms do not look ahead to the impact of changes in input prices on downstream prices, a common simplification in recent empirical work on bargaining. Our interest is in what can be said about vertical merger effects in a more traditional setting where contracting firms consider how changes in input prices affect downstream decisions and where nonlinear contracts are feasible. We also provide a method for determining the net effects of vertical mergers in settings with both price and investment decisions that does not require full structural estimation.

The remainder of the paper is organized as follows. Section II presents the model. Section III formally characterizes the equilibrium prices and investments with and without the merger. Section IV examines the effects of the merger. Section V applies the linear-quadratic model to the merger of Arm and NVIDIA. Section VI discusses extensions, and Section VII concludes.

II. A Model of Bilateral Contracting with Upstream and Downstream Investment

An upstream monopolist produces an input that may be used by one or two downstream sellers to produce a product. Downstream firm 1 (the potential merger partner) is an incumbent already in the market, and downstream firm 2 may enter the market. The final demand for product i is $D_i(p_1, p_2, e_u, e_1, e_2, \gamma)$ $(i = 1, 2)$ where p_i is the price of product i; e_u and e_i are investment efforts by the upstream firm and downstream firm i, respectively; and γ is a parameter that measures the degree of price substitution between products.[12] We make the standard assumptions that firm i's demand is continuously differentiable, strictly decreasing in own price, and increasing in the rival's price with $\partial D_i/\partial p_j < |\partial D_i/\partial p_i|$. When $\gamma = 0$, the products are independent in demand, with $(\partial D_i/\partial p_j)|_{\gamma=0} = (\partial D_i/\partial e_j)|_{\gamma=0} = 0$. As γ increases, the products become closer substitutes in the sense that $\partial^2 D_i/\partial p_j \partial\gamma > 0$, and they become perfect substitutes in the limit as γ approaches infinity, i.e., $\lim_{\gamma\to\infty} \partial D_i/\partial p_j = \infty$.

The upstream firm produces the input at constant marginal cost c. Downstream firms use the input in fixed proportions with another competitively-supplied input to produce output, and one unit of each input is required to produce each unit of output. Downstream firm i's marginal cost is the price it pays for the other input, v, plus the wholesale price it pays for the upstream product, w_i. Downstream firm 2 must also make a sunk investment $K > 0$ to enter the market.

[12] If there is an upstream competitor that sells through other downstream firms or is integrated into the downstream market, D_1 and D_2 should be interpreted as residual demands given the final prices of the upstream competitor's products.

We assume that downstream firm i's demand is strictly increasing in both its own, and the upstream firm's investment, and may increase or decrease in downstream firm j's investment. The upstream and downstream firms together comprise an ecosystem of producers in a vertical channel in which upstream and downstream investments are complementary: An investment by the upstream firm increases the demands for downstream firms' products; an investment by either downstream firm increases demand for the upstream product. An investment by either downstream firm may increase (positive spillovers) or decrease (business stealing) the demand for the other downstream firm's product.

The upstream firm's investment cost is $I_u(e_u)$ and downstream firm i's investment cost is $I_i(e_i)$. We assume investment costs are continuously differentiable, increasing ($I_k'(e_k) > 0,\ k = u, 1, 2$), and convex ($I_k''(e_k) > 0,\ k = u, 1, 2$).

Investments, contracts, and downstream prices are determined in the following game:

Stage 1: The upstream firm and downstream firm 1 decide whether to merge.

Stage 2: Downstream firm 2 decides whether to make sunk investment K to enter the downstream market.

Stage 3: The upstream firm and downstream firms that have entered simultaneously choose their investments, I_k $(k = u, 1, 2)$, which are assumed sunk.

Stage 4: The upstream firm and each downstream firm that has entered simultaneously negotiate two-part tariff contracts of the form

$$T_i(q_i) = \begin{cases} F_i + w_i q_i, & \text{if } q_i > 0,\ i = 1, 2, \\ 0 & \text{otherwise} \end{cases}$$

where F_i, w_i, and q_i are respectively the fixed fee, wholesale price, and quantity of downstream firm i $(i = 1, 2)$.[13]

Stage 5: Downstream firms simultaneously choose prices.

All firms observe prior decisions before negotiating their contracts, but downstream firms do not observe the terms of their rivals' contracts when choosing prices.

The bargaining equilibrium concept is simultaneous asymmetric Nash bargaining with unobservable contracts, which means that *changes* in downstream firm i's wholesale price do not affect

[13] There is no loss of generality in assuming that $T_i(q_i)$ is a two-part tariff, but this assumption simplifies the exposition.

firm j's downstream price, although equilibrium downstream prices are Bertrand equilibrium prices given the equilibrium wholesale prices. This equilibrium concept is equivalent to a contract equilibrium (with unobservable contracts)[14] in which fixed fees are negotiated through simultaneous asymmetric Nash bargaining according to firms' bargaining strengths. Firms are sequentially rational and look ahead at each stage of the game to future sequentially rational play. We focus on interior equilibria where all firms in the market produce positive quantities and make non-zero investments, and we assume that the pre-merger equilibrium is symmetric.

The timing of the game is designed to capture the investment effects of bilateral contracting externalities and hold-up that exist both pre- and post-merger. In particular, firms cannot write complete contracts specifying their investments, but instead look ahead when making investments to the financial terms and pricing strategies they expect given the investments and subsequent bilateral negotiations and pricing.

III. Equilibrium Prices, Contracts, and Investments

A. No Merger

Suppose the upstream firm and downstream firm 1 choose not to merge. Following the usual procedure, we determine equilibrium prices and investments in the continuation game (conditional on no merger) by starting with the last stage of the game and working backward. Let $R_i(w_i, p_{-i}, \mathbf{e}, \gamma)$ $(i = 1, 2)$ be the firm i's best response to the rival firm's price p_{-i} given the investment vector $\mathbf{e}$. Let $p_i^B(\mathbf{w}, \mathbf{e}, \gamma)$ $(i = 1, 2)$ be the Bertrand equilibrium prices such that $p_i^B = R_i(w_i, p_{-i}^B, \mathbf{e}, \gamma)$ $(i = 1, 2)$ given the wholesale price vector $\mathbf{w}$ and investment vector $\mathbf{e}$. We assume that $\mathbf{p^B}(\mathbf{w}, \mathbf{e}, \gamma)$ exists and is unique.

Working backward to contract negotiations, the variable profits of the upstream firm and downstream firm i (gross of fixed fees and investment costs) are respectively

$$\pi_u(\mathbf{w}, \mathbf{e}, \gamma) = (w_1 - c) D_1\big(\mathbf{p^B}(\mathbf{w}, \mathbf{e}, \gamma), \mathbf{e}, \gamma\big) + (w_2 - c) D_2\big(\mathbf{p^B}(\mathbf{w}, \mathbf{e}, \gamma), \mathbf{e}, \gamma\big), \tag{1}$$

and

$$\pi_i(\mathbf{w}, \mathbf{e}, \gamma) = \big(p_i^B(\mathbf{w}, \mathbf{e}, \gamma)\big) - w_i - v\big) D_i\big(\mathbf{p^B}(\mathbf{w}, \mathbf{e}, \gamma), \mathbf{e}, \gamma\big)\big) \quad i = 1, 2. \tag{2}$$

The upstream firm's payoff in Nash bargaining with firm i is $\pi_u + F_1 + F_2$, and the downstream

[14] A contract equilibrium (Cremer and Riordan, 1987) is a set of contracts that are bilaterally optimal for each bargaining pair given the upstream firm's contract with the other downstream firm. The "with unobservable contracts" qualifier was added by O'Brien and Shaffer (1992) to reflect the idea that when contracts are private information, downstream firms cannot respond to changes in a rival firm's contracts because they do not observe them.

firm i's payoff is $\pi_i - F_i$.[15] These payoffs exclude investment costs because they are assumed to be sunk costs when negotiations occur.

Downstream firm i's disagreement profit in negotiations is zero because it requires the upstream firm's product to make sales. When both downstream firms are in the market, the upstream firm's disagreement profit in negotiations with firm i is the profit it earns when firm j is a downstream monopolist, $\pi_u(\mathbf{w_{-i}}, \mathbf{e}, \gamma)$, where the notation $\mathbf{w_{-i}}$ indicates that $w_i = \infty$ and firm i's downstream price is at its choke value, and w_j is firm j's equilibrium wholesale price. The Nash product in negotiations between the upstream firm and downstream firm i is the product of their payoffs net of disagreement profits, weighted by their bargaining weights:

$$NP_i^2 = \left[\pi_u(\mathbf{w}, \mathbf{e}, \gamma) + F_i + F_j - \left(\pi_u(\mathbf{w_{-i}}, \mathbf{e}, \gamma) + F_j\right)\right]^{\alpha_i} \left[\pi_i(\mathbf{w}, \mathbf{e}, \gamma) - F_i\right]^{(1-\alpha_i)} \tag{3}$$

where α_i is the upstream firm's bargaining weight in negotiations with downstream firm i and the superscript '2' indicates that both downstream firms have entered. When only downstream firm i has entered, the disagreement profits of both firms are zero, and the Nash product is

$$NP_i^1 = \left[\pi_u(\mathbf{w_{-j}}, \mathbf{e_{-j}}, \gamma) + F_i\right]^{\alpha_i} \left[\pi_i(\mathbf{w_{-j}}, \mathbf{e_{-j}}, \gamma) - F_i\right]^{(1-\alpha_i)} \tag{4}$$

where $\mathbf{e_{-j}}$ indicates that $e_j = 0$, i.e., firm j does not make an investment if it does not enter.

It will be helpful in developing intuition to compare the incentives of unintegrated firms with those of a fully integrated firm that owns all three firms. Define joint variable profits as $\pi^J(\mathbf{w}, \mathbf{e}, \gamma) = \pi_u(\mathbf{w}, \mathbf{e}, \gamma) + \pi_1(\mathbf{w}, \mathbf{e}, \gamma) + \pi_2(\mathbf{w}, \mathbf{e}, \gamma)$. Let $\mathbf{w^I}$ and $\mathbf{e^I}$ be the wholesale price and investment vectors that maximize joint profits, and define $p_i^I = p_i^B(\mathbf{w^I}, \mathbf{e^I}, \gamma)$ as the fully integrated downstream price of firm i ($i = 1, 2$). Our results rely on combinations of the following assumptions.

Assumption 1. All variable profits are twice continuously differentiable in all arguments;

$$\pi_k(\mathbf{w}, \mathbf{e}, \gamma) - I_k(e_k) \text{ is strictly quasiconcave in } e_k, \; k = u, 1, 2;$$

$$\pi_u(c, w_2, \mathbf{e}, \gamma) + \pi_1(c, w_2, \mathbf{e}, \gamma) - I_u(e_u) - I_1(e_1) \text{is strictly quasiconcave in } (e_u, e_1).$$

Assumption 2. Investment increases variable profits:

$$\frac{\partial \pi_k(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k} > 0, \quad \frac{\partial \pi_k(\mathbf{w_{-i}}, \mathbf{e_{-i}}, \gamma)}{\partial e_k} > 0, \quad \frac{\partial}{\partial e_k}\left(\pi_u(\mathbf{w}, \mathbf{e}, \gamma) - \pi_u(\mathbf{w_{-i}}, \mathbf{e}, \gamma)\right) \geq 0, \quad k = u, 1, 2.$$

[15] In negotiations between the upstream firm an downstream firm i, rival firm $-i$ does not observe firm i's wholesale price. Thus, in Nash bargaining between the upstream firm and downstream firm i, *changes* in w_i considered during negotiations are reflected in movements along firm i's best response function $R_i(w_i, p_{-i}, \mathbf{e}, \gamma)$ rather than movements along $p_i^B(\mathbf{w}, \mathbf{e}, \gamma)$, and firm $-i$'s price held fixed at $p_{-i}^B(\mathbf{w}, \mathbf{e}, \gamma)$. Because we only need to evaluate π_u and π_i at equilibrium values in what follows, we do not make this distinction explicit for brevity of notation, but it should be understood.

Assumption 3. Marginal investment returns rise with changes in $\mathbf{w}$ that increase variable joint profits:

$$\frac{\partial \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial w_i} > 0 \Longleftrightarrow \frac{\partial^2 \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k \partial w_i} > 0, \quad k = u, 1, 2; \; i = 1, 2.$$

$$\frac{\partial \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial w_i} < 0 \Longleftrightarrow \frac{\partial^2 \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k \partial w_i} < 0. \quad k = u, 1, 2; \; i = 1, 2.$$

Assumption 4.

(A) Upstream and downstream investments are strategic complements:

$$\frac{\partial^2 \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_u \partial e_k} \geq 0, \quad \frac{\partial^2 \pi^J(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k \partial e_u} \geq 0, \quad k = 1, 2,$$

$$\frac{\partial^2 \pi_u(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_u \partial e_k} \geq 0, \quad \frac{\partial^2 \pi_k(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k \partial e_u} \geq 0, \quad k = 1, 2.$$

(B) Downstream investments are strongly complementary:

$$\frac{\partial \pi_k(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_j} \geq 0, \quad \frac{\partial^2 \pi_k(\mathbf{w}, \mathbf{e}, \gamma)}{\partial e_k \partial e_j} \geq 0, \quad k = 1, 2, \; j \neq k.$$

As noted above, this assumption is required for only Proposition 1.

Assumption 5. Holding $\mathbf{w}$ fixed, an increase in investment I_k $(k = u, 1, 2)$ increases consumer surplus.

Assumption 1 is a technical assumption that ensures that the system of equations defining the equilibrium is continuously differentiable and that each firm's profit-maximizing investment is unique conditional on wholesale prices.

Assumption 2 says that an increase in a firm's own investment strictly increases its variable profit and weakly increases the upstream firm's incremental variable profit from reaching agreement with a second downstream. Note that this assumption does not follow immediately from the assumption that D_i is strictly increasing in own- and upstream investment because all Bertrand equilibrium prices generally adjust in response to any change in investment. This assumption rules out the possibility that (i) an increase in investment by a downstream firm causes prices to change in a way that lowers the profits of the investing firm or the upstream firm, and (ii) an increase in investment by any one of the firms reduces upstream profits.

Assumption 3 says that when the wholesale price is less than the price that induces the fully integrated downstream price as the outcome of downstream competition, an increase in the wholesale price, which increases downstream prices and the fully integrated margin $p_i - c - v$, weakly increases each investment made by a firm that captures all joint profits. Intuitively, the joint marginal benefit from investment that shifts demand at given prices is the joint margin earned on the increment in sales due to the investment. Assumption 3 means that the greater the fully integrated margin (which rises with w for all $w \in [c, w^I]$), the greater the joint marginal benefit of each investment after accounting for how prices adjust in Bertrand price equilibrium.

Assumption 4(A)—strategic complementarity of upstream and downstream investments—is a natural assumption for environments where investments by upstream and downstream firms are complementary. The idea is that investment by a member of the ecosystem becomes more profitable when other complementors are also investing more. Assumption 4(B)—strong complementarity of downstream investments[16]—holds when downstream investments are complements in the sense that an investment by firm i benefits firm j, and when they are also strategic complements, such that an increase in investment by firm i causes firm j to increase its investment. Strong complementarity is a natural assumption when investment creates spillovers that increase the value of the ecosystem that sits under the upstream firm's product.

Assumption 5 ensures that after downstream prices adjust in response to an increase in one or more investments, consumers are better off. This assumption embodies more than the assumption that consumer surplus is increasing in investments because equilibrium prices general adjust in response to changes in investment.

A..1 Equilibrium When Both Downstream Firms Enter

Suppose first that both downstream firms have entered. It is well-known that the pre-merger contract equilibrium with private information yields wholesale prices equal to marginal cost: $w_1 = w_2 = c$.[17] This result is driven by the assumptions that contracts are determined bilaterally and are not observed by rivals. Specifically, because negotiations are bilateral, the wholesale price in each bilateral negotiation is chosen to maximize bilateral profits rather than joint profits. Each bargaining pair ignores the negative externality inflicted on the other downstream firm from lowering their wholesale price, leading to wholesale prices below the level that would induce the fully integrated downstream prices as the outcome of Bertrand competition. To see why wholesale prices fall all the way to marginal cost, observe that when $w_j = c$, the bilateral profits of the upstream

[16]We are borrowing this term from Spence (1976), who defined products that are both complements and strategic complements as strongly complementary. See Section 3 of his paper.

[17]See O'Brien and Shaffer (1992) and Rey and Tirole (2007). Although we focus on two-part tariffs for simplicity, there is no loss of generality in doing so. For example, the same outcome would arise if firms agreed to point contracts (T_i, q_i) where T_i is the total transfer and q_i is quantity. In this case, the marginal "shadow" price (the wholesale price that would induce the downstream firm to choose quantity q_i) would equal marginal cost, and the average price T_i/q_i (which exceeds marginal cost) would simply divide surplus.

firm and firm i are the same as they would be if they were an integrated Bertrand competitor of firm j. Moreover, they cannot use their wholesale price to induce softer competition from firm j because firm j does not observe their contract.[18] Thus, to induce downstream firm i to maximize their bilateral profit, they transfer the input at marginal cost so as not to distort downstream firm i's pricing decision. This is true for both bargaining pairs and leads to marginal cost input pricing. Imposing the wholesale prices $w_1 = w_2 = c$, the equilibrium downstream prices are given by $p_i^B(\mathbf{c}, \mathbf{e}, \gamma)$, where $\mathbf{c} = (c, c)$.

Given that wholesale prices equal marginal cost, the negotiated fixed fees maximize the Nash product NP_i^2 given the firms' investment decisions and expectations that downstream prices will be $p_i^B(\mathbf{c}, \mathbf{e}, \gamma)$ $(i = 1, 2)$. Straightforward calculations show that the fixed fee that maximizes the Nash product is

$$\begin{aligned} F_i^B(\mathbf{e}, \gamma) &= \alpha_i \pi_i(\mathbf{c}, \mathbf{e}, \gamma) - (1 - \alpha_i)\big(\pi_u(\mathbf{c}, \mathbf{e}, \gamma) - \pi_u(\mathbf{c_{-i}}, \mathbf{e}, \gamma)\big) \\ &= \alpha_i \pi_i(\mathbf{c}, \mathbf{e}, \gamma), \quad i = 1, 2 \end{aligned} \tag{5}$$

where the second equality follows from the fact that $\mathbf{w} = \mathbf{c}$ and thus $\pi_u = 0$.

Proceeding further backward to the investment stage, each firm chooses its investment to maximize its total profits (variable profit minus investment costs) with the expectation that wholesale prices will equal marginal cost, downstream prices will be $p_i^B(\mathbf{c}, \mathbf{e}, \gamma)$ $(i = 1, 2)$, and fixed fees will be $F_i^B(\mathbf{e}, \gamma)$ $(i = 1, 2)$:

$$\max_{e_u} \ \pi_u(\mathbf{w}, \mathbf{e}, \gamma) + F_1^B(\mathbf{e}, \gamma) + F_2^B(\mathbf{e}, \gamma) - I_u(e_u), \tag{6}$$

$$\max_{e_i} \ \pi_i(\mathbf{w}, \mathbf{e}, \gamma) - F_i^B(\mathbf{e}, \gamma) - I_i(e_i) \ \ i = 1, 2. \tag{7}$$

Denote the simultaneous solution to (6) and (7) as $\mathbf{e}^{**}(\gamma)$ where two asterisks indicate the solution when both firms have entered and there is no merger. Denote the equilibrium wholesale and downstream prices as $\mathbf{w}^{**} = \mathbf{c}$ and $\mathbf{p}^{**}(\gamma)$.

Finally, in deciding whether to enter the market, downstream firms look ahead to their expected profits. Both downstream firms enter if and only if

$$\pi_i(\mathbf{c}, \mathbf{e}^{**}(\gamma), \gamma) - F_i^B(\mathbf{e}^{**}(\gamma), \gamma) - I_i(e_i^{**}(\gamma)) - K \geq 0, \ \ i = 1, 2. \tag{8}$$

Otherwise no equilibrium exists in which both firms enter.

[18] If contracts were bilaterally negotiated but observed by rivals, wholesale prices would still be lower than w^I but higher than marginal cost because the upstream firm and downstream firm i would derive some benefit to inducing softer competition from firm j.

A..2 Equilibrium When One Downstream Firm Enters

If only one downstream firm enters, say firm i, the procedure for finding the equilibrium price and upstream investment for the continuation game is similar to the procedure when two firms enter. Because two-part tariffs are feasible, the optimal contract under bilateral monopoly specifies a wholesale price equal to marginal cost, and the downstream price is $p_i^B(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma)$. The fixed fee that divides profits is given by the Nash bargaining solution when only firm i is active. Straightforward calculations show that the equilibrium fixed fee is

$$\begin{aligned} F_i^B(\mathbf{e_{-j}}, \gamma) &= \alpha_i \pi_i(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma) - (1-\alpha_i)\pi_u(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma), \\ &= \alpha_i \pi_i(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma), \quad i = 1, 2. \end{aligned} \tag{9}$$

Proceeding backward, the upstream firm and the active downstream firm i will choose their investments to solve

$$\max_{e_u} \pi_u(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma) + F_i^B(\mathbf{e_{-j}}, \gamma) - I_u(e_u), \tag{10}$$

$$\max_{e_i} \pi_i(\mathbf{c_{-j}}, \mathbf{e_{-j}}, \gamma) - F_i^B(\mathbf{e_{-j}}, \gamma) - I_i(e_i). \tag{11}$$

Denote the equilibrium investments and downstream price when only firm i has entered as $\mathbf{e}^*(\gamma)$ and $p_i^*(\gamma)$.

Lastly, only one firm will enter if and only if its profit as a duopolist is negative (i.e., if condition (8) is not satisfied) and its profit as a monopolist is positive:

$$\pi_i(\mathbf{c}, \mathbf{e}^*(\gamma), \gamma) - F_i^B(\mathbf{e}^*(\gamma), \gamma) - I_i(e_i^*(\gamma)) - K \geq 0. \tag{12}$$

B. A Baseline Result

In this model, with private bilateral bargaining, there is no pre-merger double marginalization. In the absence of investment considerations, the model would predict that a vertical merger would raise prices and harm consumers except in the extreme case where the products are independent in demand. Critically, however, the following proposition establishes that in the absence of a merger, all investments are less than the investments a fully integrated firm would make under strategic complementarity. This creates the potential for a vertical merger to enhance competition even without eliminating double marginalization, and the proof of the proposition provides an intuitive understanding of the role of vertical integration in our model.

Proposition 1. *Under Assumptions 1-5, a fully integrated firm chooses higher levels of all investments than unintegrated firms.*

Proof: Substituting F_i^B from (5) into the investment profit objectives (6) and (7) and rearranging, the maximization problems that characterize investment choices in the absence of integration can be written as follows:

$$\max_{e_u} \;\; \pi^J - I_u(e_u) - [(1-\alpha_1)\pi_1 + (1-\alpha_2)\pi_2] \tag{13}$$

and

$$\max_{e_i} \;\; \pi^J - I_i(e_i) - [(1-\alpha_i)\pi_j + \alpha_i \pi^J], \;\; i = 1, 2, \tag{14}$$

where $\pi_{u,-i} \equiv \pi_u(\mathbf{w_{-i}}, \mathbf{e})$, and we have used that $\pi_u = \pi_{u,-i} = 0$ in equilibrium since $\mathbf{w} = \mathbf{c}$. The first order conditions are

$$\underbrace{\left[\frac{\partial \pi^J}{\partial e_u} - I'_u\right]\Big|_{w=c}}_{\text{Marginal joint return}} - \underbrace{\left[(1-\alpha_1)\frac{\partial \pi_1}{\partial e_u} + (1-\alpha_2)\frac{\partial \pi_2}{\partial e_u}\right]\Big|_{w=c}}_{\text{Distortion due to hold up}} = 0 \tag{15}$$

$$\underbrace{\left[\frac{\partial \pi^J}{\partial e_i} - I'_i\right]\Big|_{w=c}}_{\text{Marginal joint return}} - \underbrace{\left[(1-\alpha_i)\frac{\partial \pi_j}{\partial e_i} + \alpha_i\frac{\partial \pi^J}{\partial e_i}\right]\Big|_{w=c}}_{\text{Distortion due to hold up}} = 0, \;\; i = 1, 2, j \neq i. \tag{16}$$

The term labeled "Marginal joint return" in each first order condition is the increment in joint variable profit from investment when wholesale prices are evaluated at c. The term labeled "Distortion due to hold up" in each condition is the wedge between joint profits and the profits of the investing firm due to the hold-up created by ex post bargaining. A fully integrated firm would choose investment such that the marginal joint return term is zero when evaluated at $w^I > c$, i.e., $\partial \pi^J(\mathbf{w^I}, \mathbf{e}, \gamma)/\partial e_k - I'_k(e_k) = 0$ $(k = u, 1, 2)$. Thus, the first order conditions in the absence of a merger differ from the optimizing conditions of a fully integrated firm in two ways: (i) the variable profits are evaluated at the wholesale price c rather than the fully integrated wholesale price w^I, and (ii) there is a distortion due to hold-up. Both factors distort investment from the fully integrated levels. That is, absent a merger, joint profit dissipation due to bilateral bargaining (which drives wholesale prices all the way down to c) and hold-up both reduce investment incentives.

To sign the investment distortion, break the effects of vertical integration into two steps as follows. Define a *w-integrated firm* as one that chooses investments as a fully integrated monopolist but sets downstream prices as two unintegrated Bertrand competitors that pay the wholesale price w. Note that a c-integrated firm has the same marginal returns from investment as unintegrated firms, but its investment choice does not include a distortion due to hold-up. A w^I-integrated firm has the same incentives as a fully integrated firm. We show that shifting from from no integration to c-integration to w^I-integration raises all investments at both steps.

Start with no integration. The distortion due to hold-up in each of (15) and (16) is positive by Assumptions 2 and 4(B). This means that the marginal joint return in each equation has to be positive to satisfy the first order condition. Because each firm's profit is strictly quasiconcave (Assumption 1), a shift to c-integration that eliminates the distortion due-to hold-up requires an increase in each e_k $(k = u, 1, 2)$ to satisfy the relevant first order condition *given* the other investments. Since investments are strategic complements, all investments rise in equilibrium. That is, a c-integrated firm chooses higher levels of all investments than unintegrated firms choose independently. The logic is that a c-integrated firm faces no hold-up, and the direct incentives to invest more due to the elimination of hold up are reinforced by strategic complementarity.

Next, raise w from c to w^I to create a w^I-integrated firm, the equivalent of a fully integrated firm. By Assumption 3, this raises the marginal return of each investment, creating an incentive to raise each investment level conditional on the others (i.e., raising w increases $\partial\pi^J/\partial e_k$, $k = u, 1, 2$, which requires an increase e_k to satisfy the relevant first order condition). Since all investments are strategic complements, all investments increase. Therefore, a w^I-integrated firm chooses higher levels of all investments than a c-integrated firm.

We have shown that a fully integrated firm will choose higher levels of all investments than a c-integrated firm, which in turn will choose higher levels of all investments than unintegrated firms. This establishes that under Assumptions 1-5, a fully integrated firm chooses higher levels of all investments than unintegrated firms. □

It should be noted that if downstream investments are strategic substitutes rather than strategic complements, a fully integrated firm will still choose higher levels of all investments than an unintegrated firm if the degree of strategic substitutability is small enough. This follows from the continuity of the equilibrium investment levels. It would be natural for downstream investments to be strategic substitutes if downstream investment steals business from rivals rather than creating positive demand spillovers.[19]

[19] In his well-known model of product selection, Spence (1976) shows if products are strongly complementary, the market outcome provides both too little entry and too little output conditional on entry. See also Vives (1999) pp. 176-177. Interpreting quantity in Spence's model as R&D expenditure (which is positively related to quantity), our finding that full integration raises investments under strong complementarity of investments is related to his finding of too little output, although our model adds the element of complementary upstream investment, and the socially optimal benchmark considered by Spence generally differs from the fully integrated outcome.

C. Vertical Merger

Now assume that the upstream firm and downstream firm 1 merge.[20]

C..1 Equilibrium When Firm 2 Enters Post-Merger

Two immediate implications of the merger are that downstream firm 1's wholesale price is automatically fixed at marginal cost, and the character of the downstream pricing game changes because the merged firm that controls p_1 may earn variable profits $(w_2 - c)D_2$ from selling the product at wholesale to firm 2. Thus, final prices are not simply $p_1^B(c, w_2, \mathbf{e}, \gamma)$ and $p_2^B(c, w_2, \mathbf{e}, \gamma)$, but rather they must be determined by using the new objective for firm 1. Firm 1's price objective is to maximize the integrated profit $\pi^{Int} = (p_1 - c - v)D_1(\mathbf{p}, \mathbf{e}, \gamma) + (w_2 - c)D_2(\mathbf{p}, \mathbf{e}, \gamma)$, while firm 2's price objective is still to maximize $\pi_2 = (p_2 - c - v)D_2(\mathbf{p}, \mathbf{e}, \gamma)$. Denote the prices that simultaneously maximize these objectives as $\mathbf{p^{Int}}(c, w_2, \mathbf{e}, \gamma)$.

To determine the equilibrium wholesale price for firm 2, note that because fixed fees are feasible, the merged firm and firm 2 will choose w_2 to maximize their *bilateral* profits looking ahead to the vector of pricing strategies $\mathbf{p^{Int}}(c, w_2, \mathbf{e}, \gamma)$. Their bilateral profits are the joint profits earned in the vertical structure,[21] so the wholesale price solves

$$\max_{w_2} \pi^J = (p_1^{Int}(c, w_2, \mathbf{e}, \gamma) - c - v)D_1\big(\mathbf{p^{Int}}(c, w_2, \mathbf{e}, \gamma), \mathbf{e}, \gamma\big) + (p_2^{Int}(c, w_2, \mathbf{e}, \gamma) - c - v)D_2\big(\mathbf{p^{Int}}(c, w_2, \mathbf{e}, \gamma), \mathbf{e}, \gamma\big). \quad (17)$$

The first order condition evaluated at $w_2 = c$ is

$$\Big\{\big[D_1 + (p_1^{Int} - c - v)\frac{\partial D_1}{\partial p_1}\big] + (p_2^{Int} - c - v)\frac{\partial D_2}{\partial p_1}\Big\}\frac{\partial p_1^{Int}}{\partial w_2} + \Big\{\big[D_2 + (p_2^{Int} - c - v)\frac{\partial D_2}{\partial p_2}\big] + (p_1^{Int} - c - v)\frac{\partial D_1}{\partial p_2}\Big\}\frac{\partial p_2^{Int}}{\partial w_2} = 0. \quad (18)$$

The terms in square brackets inside the curly braces are the first order conditions profit maximization for firms 1 and 2 prior to the merger when $w_1 = w_2 = c$. The other terms are positive if downstream products are substitutes and strategic complements in price. Thus, the post-merger wholesale price exceeds marginal cost unless products are independent, in which case it chooses a

[20] A merger is always profitable in this model unless there is some unmodelled transaction cost.

[21] Although the merged firm's objective now consists of all joint profit that exists, it is not able to replicate the fully integrated outcome because the constraint that $w_1 = c$ for the merged firm distorts marginal incentives in setting w_2. If the merged firm could somehow commit to pretending that w_1 is the wholesale price that induces the fully integrated outcome, it would negotiate a wholesale price with firm 2 that would achieve the fully integrated outcome. But it is not possible to make this commitment, and this prevents the merged firm from achieving the fully integrated outcome.

wholesale price equal to marginal cost. Denote the profit maximizing wholesale price when both firms have entered as $w_2^{Int}(\mathbf{e},\gamma)$.

Proceeding backward, the fixed fee that maximizes the Nash product is

$$F_2^{Int}(\mathbf{e},\gamma)=\alpha_2\pi_2\big(c,w_2^{Int}(\mathbf{e},\gamma),\mathbf{e},\gamma\big)-(1-\alpha_2)\big[\pi^{Int}\big(c,w_2^{Int}(\mathbf{e},\gamma),\mathbf{e},\gamma\big)-\pi^{Int}(\mathbf{c_{-2}},\mathbf{e_{-2}},\gamma)\big]. \quad (19)$$

The equilibrium investment levels now solve

$$\max_{e_u,e_1}\ \pi^{Int}(c,w_2,\mathbf{e},\gamma)+F_2^{Int}(\mathbf{e},\gamma)-I_u(e_u)-I_1(e_1), \quad (20)$$

$$\max_{e_2}\ \pi_2(c,w_2,\mathbf{e},\gamma)-F_2^{Int}(\mathbf{e},\gamma)-I_2(e_2). \quad (21)$$

Denote the equilibrium wholesale price, downstream prices, and investment levels as $w_2^{++}(\gamma)$, $\mathbf{p}^{++}(\gamma)$, and $\mathbf{e}^{++}(\gamma)$.

C..2 Equilibrium When Firm 2 Does Not Enter

If firm 2 would not enter in response to the merger, the merged firm would choose the one-firm-monopoly price and investment levels, which solve

$$\max_{p_1,e_u,e_1}(p_1-c-v)D_1(p_1,\infty,e_u,e_1,0,\gamma)-I_u(e_u)-I_1(e_1). \quad (22)$$

Denote the equilibrium retail price and investment levels as $p_1^+(\gamma)$, $I_u^+(\gamma)$, and $I_1^+(\gamma)$.

IV. Merger Effects

A. Main Results

The following proposition is the main result of the paper.

Proposition 2. *Under Assumptions 1-3, 4(A), and 5, there exist critical values of the substitution parameter γ^L and γ^H such that the merger strictly increases consumer welfare for all $\gamma<\gamma^L$ and for all $\gamma>\gamma^H$. That is, if the degree of product substitution is low enough or high enough, the merger is pro-competitive. For intermediate degrees of product substitution, the competitive effects of the merger depend on the other parameters.*

Proof: We prove the proposition by considering the limiting cases of independent products ($\gamma=0$) and perfect substitutes ($\gamma\to\infty$) and using continuity of the equilibrium choices in γ to establish the Proposition.

Consider first the case where downstream firms' demands are independent ($\gamma = 0$). In this case, using the fact that $w_1 = w_2 = c$ and substituting the fixed fees F_i^B into the objectives in (6) and (7), the pre-merger investment choices solve (with obvious labels):

$$\text{(PreU)} \ \max_{e_u} \ \alpha_1\pi_1 + \alpha_2\pi_2 - I_u, \quad \text{(Pre1)} \ \max_{e_1} \ (1-\alpha_1)\pi_1 - I_1, \quad \text{(Pre2)} \ \max_{e_2} \ (1-\alpha_2)\pi_2 - I_2$$

In the post-merger situation with independent products, it is also the case that $w_1 = w_2 = c$. Using this fact and substituting the fixed fee F_2^{Int} into (20) and (21), the post-merger maximization problems solve

$$\text{(PostU)} \ \max_{e_u} \ \pi_1 + \alpha_2\pi_2 - I_u, \quad \text{(Post1)} \ \max_{e_1} \ \pi_1 + \alpha_2\pi_2 - I_1, \quad \text{(Post2)} \ \max_{e_2} \ (1-\alpha_2)\pi_2 - I_2.$$

Comparing the pre- and post-merger objectives for each investment choice shows that each post-merger objective includes the variable profit in the corresponding pre-merger objective plus additional components of profits. Since π_i is increasing in e_u and e_i (Assumption 2) and is independent of e_j when $\gamma = 0$, and the objectives are strictly quasiconcave (Assumption 1), the investment level that maximizes each post-merger objective exceeds the investment level that maximizes the corresponding pre-merger objective, holding other investments fixed. Since upstream and downstream investments are strategic complements, all investments increase. By continuity, there is some $\gamma^L > 0$ (which could include all values of γ) such that the merger increases all investments for all $\gamma < \gamma^L$. By Assumption 5, the merger increases consumer welfare for these values of γ.

Next, suppose that downstream firms are nearly perfect substitutes ($\gamma \to \infty$). In this case, when both downstream firms are in the market, pre-merger downstream prices are close to marginal cost, and variable profits are small, approaching zero in the limit as ($\gamma \to \infty$). For sufficiently large γ, firm 2 will not enter with or without the merger because its entry cost K will exceed its profit. In this case, an effect of the merger is that merged firms internalize the joint profits from their investments, which expands investment (straightforward to demonstrate using analysis similar to that above) and increases consumer welfare by Assumption 5.[22] This establishes that there is some γ^H (potentially 0) such that the merger increases consumer welfare for all $\gamma > \gamma^H$.

For intermediate ranges of differentiation, the merger can have ambiguous effects, depending on the other parameters (investment costs, bargaining weights, and downstream investment spillovers).

[22] A second possible effect of the merger in this case is that the additional investment induced by the merger could make it profitable for firm 2 to enter the market by shifting out firm 2's demand. This would benefit firm 2 (see Corollary 1 below). It is possible that overall investment would expand and consumer welfare would rise as well, but this is not a necessary implication of Assumptions 1-5.

This is illustrated in the examples in the next section.[23] □

A corollary to Proposition 2—that the unintegrated rival benefits from the merger if differentiation is either sufficiently strong or weak—arises because the increases in investment due to the merger spillover to the unintegrated downstream firm. In particular, if demands are nearly independent, the merger does not change the wholesale price of firm 2, but firm 2 benefits from the merger because the merging firms increase investment after mitigating hold-up, and other investments rise because upstream and downstream investments are strategic complements. And when products are sufficiently close substitutes, firm 2 stays out of the market in the absence of the merger and may enter the market post-merger due to the increase in investment. The result is that the unintegrated rival benefits from the merger if differentiation is either sufficiently strong or weak.

Corollary 1. *Under Assumptions 1-3, and 4(A) there exist critical values of the substitution parameter $\gamma^{L'}$ and $\gamma^{H'}$ such that the merger weakly increases the unintegrated rivals's profit for all $\gamma < \gamma^{L'}$ and all $\gamma > \gamma^{H'}$. That is, if the degree of product substitution is low enough or high enough, the merger benefits the unintegrated downstream firm. For intermediate degrees of product substitution, the effect of the merger on the unintegrated downstream firm depends on the other parameters.*

B. Parameterized Example

In this section we examine a parametric example that illustrates Proposition 2 and Corollary 1 and shows how merger effects depend on the parameters. We use the following Levitan-Shubik demand form modified to include investment:

$$D_i(p_1, p_2, e_u, e_1, e_2, \gamma, \rho) = 1 + e_u + e_i + \rho e_j - p_i + \gamma(p_j - p_i), \quad i = 1, 2; \quad j \neq i$$

where ρ may be positive or negative. We assume that investment costs are given by $I_u(e_u) = \beta e_u^2$ and $I_i(e_i) = \theta e_i^2$, $(i = 1, 2)$.

[23] When differentiation is strong and price effects are relatively unimportant, the result that consumers benefit from the merger is driven by the elimination of hold-up due to ex post bargaining. A reasonable question is whether this benefit would disappear if contracts were written prior to investment, which would eliminate hold-up. Because investment involves double moral hazard, a contract written prior to investments would adjust the wholesale price to balance the benefits of more upstream investment from higher wholesale margins against the cost of lower investment from lower downstream margins, and both investments would be suboptimal. Thus, the effects of vertical merger would still involve a tradeoff between enhanced investment incentives and price effects. Moreover, unless downstream investment spillovers are large, this balancing would specify positive wholesale margins. (For why downstream spillovers can lead to wholesale prices below marginal cost, see Gabrielsen and Johanson (2017), who examine a model with downstream investment and spillovers and no upstream investment.) Thus, there would be benefits from eliminating double marginalization. The analysis of the effect of vertical mergers in the double moral hazard environment is a topic for further research.

Unless indicated otherwise, all the examples below assume zero production costs ($c = v = 0$), symmetric Nash bargaining ($\alpha_1 = \alpha_2 = .5$), zero downstream investment spillovers ($\rho = 0$), and that the downstream entry cost K is low enough that both downstream firms are in the market both pre- and post-merger.[24]

B..1 The Role of Product Differentiation and Investment Costs when Both Firms Enter

The four panels in Figure 1 show how consumer surplus varies with the merger, the substitution parameter γ, and the investment cost parameters β and θ assuming, for illustration, that $\beta = \theta$.

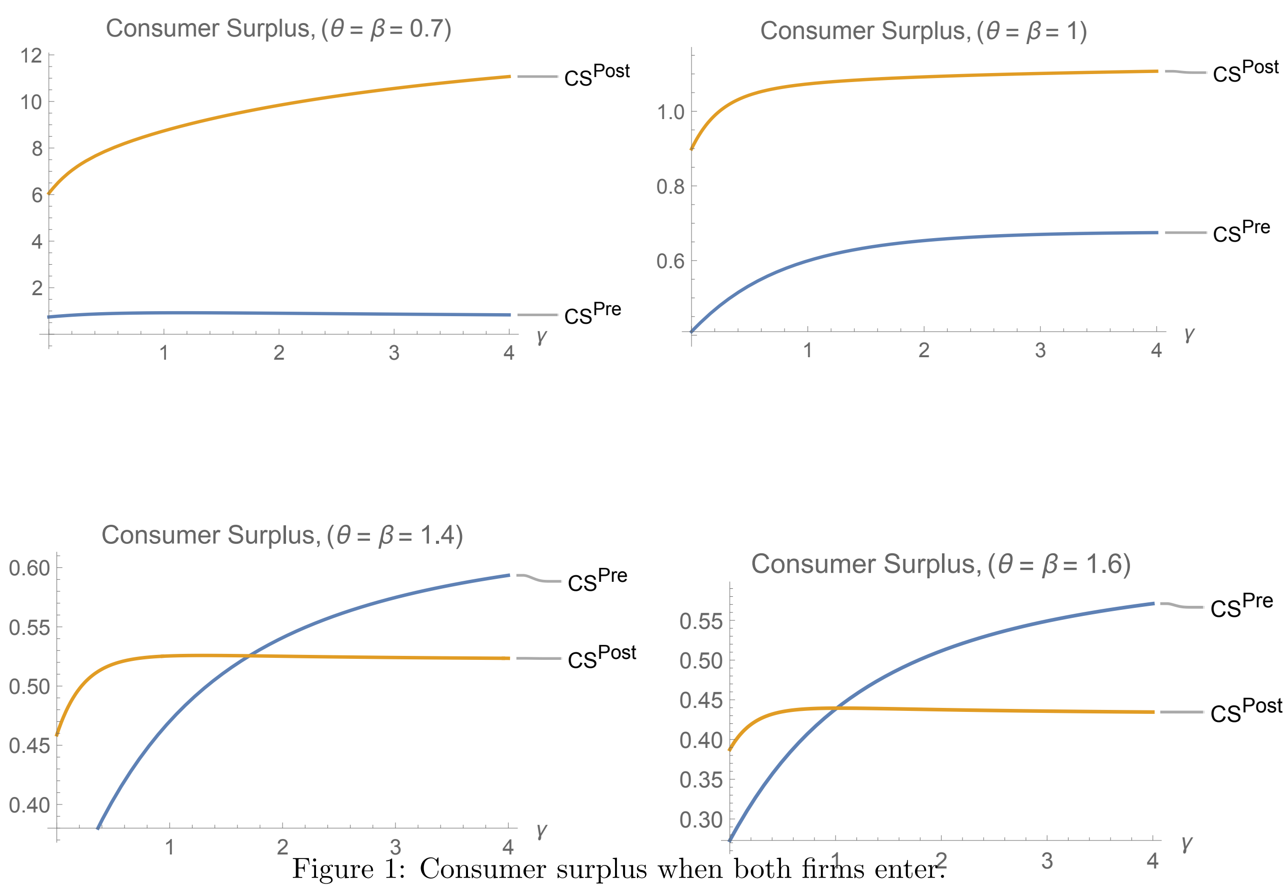

Figure 1: Consumer surplus when both firms enter.

CS^{Pre} and CS^{Post} are the equililbrium levels of pre-merger and post-merger consumer surplus, respectively. Note first that regardless of the investment cost, the merger increases consumer surplus when downstream demands are independent ($\gamma = 0$). This confirms the conclusion in Proposition 2 that the merger benefits consumers when the downstream firms are sufficiently differentiated.

[24]The formulas for equilibrium values of prices, investment, profits, and consumer surplus in this framework are tedious and were worked out in Mathematica. The code is available upon request.

Second, the panels show that if investment costs are sufficiently small (the upper two panels), the merger benefits consumers for all values of the substitution parameter. That is, if investment costs are low enough, the investment benefits of the merger outweigh price effects regardless of the degree of product differentiation and thus the size of bilateral contracting externalities, which increase with γ. (It can be shown that the the rankings of pre-merger and post-merger consumer surplus remain the same as γ grows beyond 4 in all four panels.)[25]

Third, if investment costs are high enough, there is critical substitution parameter such that the merger reduces consumer surplus for all substitution parameters that exceed the critical value. This occurs because when investment costs are high enough, the investment benefits from the merger are insufficient to offset the relatively large price increases due to mitigating large bilateral contracting externalities, which increase with γ.

B..2 Effects on the Unintegrated Rival and Entry

Figure 1 assumes that both firms enter the market for all of the parameters and thus does not illustrate the effects of the merger on entry. The panels in Figure 2 show how the net profits of downstream firm 2 (the unintegrated rival) excluding the fixed entry cost vary with the merger, the substitution parameter, and investment costs. This allows determining whether there are ranges of entry costs such that the merger encourages or discourages entry.

Observe first that if downstream firms are independent ($\gamma = 0$), the unintegrated rival always benefits from the merger ($\pi_2^{Post} > \pi_2^{Pre}$). This is consistent with Corollary 1, which indicates that if the degree of product differentiation is sufficiently high, the unintegrated rival benefits from the merger.

Second, for values of γ such that $\pi_2^{Post} > \pi_2^{Pre}$, there exist entry cost values intermediate between π_2^{Post} and π_2^{Pre} such that only one downstream firm would enter in the absence of the merger but both would enter after the merger. This is more likely the greater the degree of product differentiation (the smaller the value of γ) and the smaller the investment cost. Observe that if investment costs are small (e.g., the upper left panel in Figure 2), the merger increases the unintegrated rival's profit by several hundred percent, which means that the range of entry costs for which the merger induces entry can be large.

[25]The lower right panel in Figure 1 might give the false impression that the merger lowers consumer surplus when the substitution parameter γ is large, in contradiction of Proposition 2. However, the graph assumes that firm 2 is in the market even though firm 2 would not enter if γ were sufficiently high. If firm 2 does not enter (because γ is large), the merger leads to an increase in investment and consumer surplus.

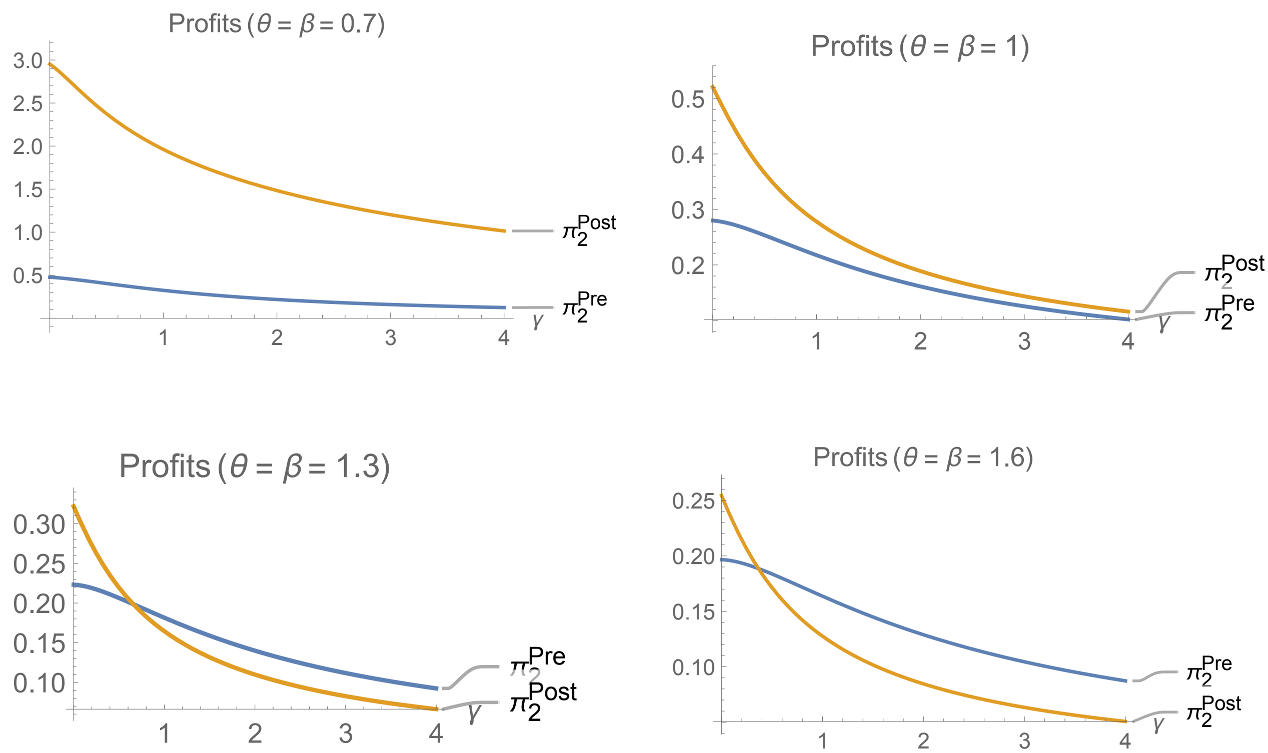

Figure 2: Unintegrated downstream rival's profits.

Third, if investment costs are high, the merger lowers the unintegrated rival's profit when the degree of product substitution is sufficiently high. The reason for this is that the investment benefits of the merger fall with investment costs while the price effects rise with the intensity of downstream competition.

Finally, because the merger can lower the unintegrated rival's profit if investment costs are high and the substitution parameter is high, the merger can induce exit.

A question one might ask is whether a complaint by the unintegrated rival provides any indication of whether the merger is likely to harm competition. Figure 3 overlays the unintegrated rivals' profits from Figure 2 with consumer surplus values from Figure 1 for the case where $\theta = \beta = 1.3$. Figure 3 shows that there exists a range of substitution parameters for which firm 2 might complain about the merger (where $\pi_2^{Pre} > \pi_2^{Post}$) but consumers would benefit from it ($CS^{Post} > CS^{Pre}$). Thus, the effect of the merger on firm 2 is not a good indicator of the effect of the merger on consumers.

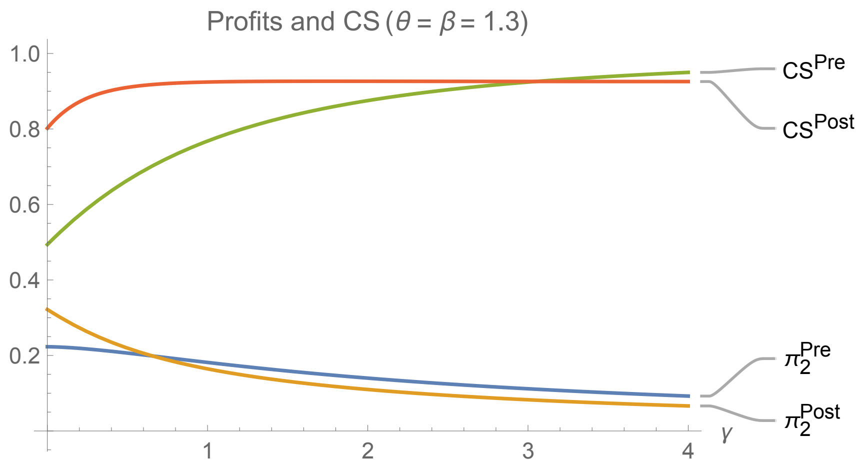

Figure 3: Unintegrated downstream rival's profits.

V. Application to the NVIDIA-Arm Merger

The theoretical ambiguity of merger effects raises the question of how to assess a merger that involves joint investments in a bilateral bargaining setting. One answer, of course, is that if there is a basis to conclude that downstream product differentiation is either very limited or extensive, then the merger increases consumer welfare per the results above. But this leaves open the question of how to evaluate a merger in the intermediate case. In this section we discuss one approach to this question and apply it to a vertical merger in the market for computer chips sold to high performance datacenters.

Our approach starts from the fact that sufficiently low investment costs imply that the merger would raise consumer surplus. To implement this in practice (given that investment costs are likely to be hard to observe/measure), we rely on the fact that, in the variant of the model with modified LS demands and quadratic investment costs, the investment-to-sales ratio in the industry increases as investment costs fall, and vice versa. Hence, we can infer the level of investment costs given the observed investment-to-sales ratio, and thus determine whether those investment costs are low enough for the merger to be welfare enhancing.

To apply this approach to the NVIDIA-Arm merger discussed in the introduction, suppose it is reasonable to assume that upstream marginal cost is zero (Arm's marginal cost of licensing CPU technology is approximately zero); the symmetric Nash Bargaining solution applies (bargaining weights in Nash bargaining depend on firms' discount rates, and there is no reason to believe that

Arm discounts the future at a different rate than downstream chip designers); and there are no downstream investment spillovers (conservative in a model with complementary investment). The remaining unknown parameters are the substitution parameter, γ, the upstream investment cost parameter, β, the downstream investment cost parameter, θ, and the downstream marginal cost due to other inputs, v.[26]

In the modified LS-quadratic investment cost framework, equilibrium prices, quanatities and investments are functions of these parameters: $\mathbf{p}(\gamma, \beta, \theta, v)$, $\mathbf{q}(\gamma, \beta, \theta, v)$, and $\mathbf{e}(\gamma, \beta, \theta, v)$. The substitution parameter consistent with equilibrium can be determined numerically as a function of the margin m_i and the other parameters using downstream first order condition $m_i = (p_i - c - v)/p_i = \frac{D_i}{-\partial D_i/\partial p_i}$ (omitting the functional dependence of endogenous variables on the parameters). That is, the downstream margin m_i and the other parameters pin down the substitution parameter, as in typical merger simulations. Define the substitution parameter that generates a downstream margin of 60 percent (NVIDIA's 10K shows a gross margin of roughly 60 percent) given the other unknown parameters as $\gamma^*(\beta, \theta, v)$.

The equilibrium pre-merger investment to sales ratio depends on the same four parameters: β, θ, v, and the degree of product subsitution $\gamma^*(\beta, \theta, v)$. To determine whether the model predicts that the merger would increase or decrease consumer surplus with this degree of product substitution, we simulated values of the pre-merger investment-to-sales ratio and the merger-induced change in consumer surplus over a grid of the unknown parameters (β, θ, v) and corresponding values of $\gamma^*(\beta, \theta, v)$.[27] The simulations cover investment-to-sales ratios that are consistent with the merger increasing and decreasing consumer surplus at an interior equilibrium with a downstream margin of 60 percent.

The simulation results indicate that for all cases in which the investment-to-sales ratio exceeds about 6%, the merger increases consumer surplus.[28] The investment-to-sales ratio of major CPU technology and chip designers in 2019 were as follows: Intel – 19%; AMD – 23%; NVIDIA – 26%; Arm – 47%.[29] Thus, the symmetric linear-quadratic model specified here predicts that a vertical

[26]The application here is approximate, as we do not have data allowing the consideration of asymmetries at the downstream level. Nevertheless, the methodology can be modified in straightforward way to account for asymmetries in other applications.

[27]We varied β and θ from 0.1 to 3.0 in increments of 0.1 and v from 0.05 to 0.25 in increments of 0.05. This generated 4500 data points, spanning cases where the merger increased and decreased consumer surplus at interior equilibria with a downstream margin of 60 percent.

[28]As a sensitivity, we expanded the grid to allow business stealing investment, with the investment-driven diversion ratio varying between 1/2 and 0 (i.e., ρ varying from -1/2 to 0). In this case, the merger increases consumer surplus for all cases in which the investment-to-sales ratio exceeds about 9%.

[29]See "The 2020 EU industrial R&D investment scoreboard," available at https://iri.jrc.ec.europa.eu/scoreboard/2020-eu-industrial-rd-investment-scoreboard, site accessed March 25, 2021.

merger in this environment would increase consumer welfare.

VI. Conclusion

We have extended the theory of vertical bilateral contracting to an environment where upstream and downstream firms make ex ante demand-enhancing investments including a fixed entry cost by a downstream entrant. In doing so, we addresses a critique in the literature that bilateral contracting models that predict harm from vertical mergers ignore the negative effect of bilateral contracting on investment incentives. We agree that if mitigation of bilateral contracting externalities is a basis for an antitrust theory of harm against vertical mergers, one must examine both the price and investment effects of that mitigation. Put simply, if failure to internalize bilateral bargaining incentives creates downward pressure on both prices (the "good" competitive effect) and investments (the "bad" competitive effect), the fact that a vertical merger eliminates or mitigates both effects must be considered in any full analysis of the effects of a vertical merger. Our model does that.

Notably, many vertical mergers are motivated by the desire to align complementary investment incentives. Where this is the case, an antitrust analysis focused only on price would miss important competitive effects of the merger. In the market for high performance computer chips, for example, R&D budgets are high because technology advances rapidly and the only way to remain competitively relevant is to make large investments on an ongoing basis. An analysis of the competitive effects of vertical mergers in that industry would badly miss the mark if it ignored the impact of the merger on investment.

To address these issues, we build a model of investment and pricing and bilateral bargaining over contract terms that explicitly incorporates bilateral contracting externalities and hold-up. A vertical merger in this environment eliminates both bilateral contracting externalities and hold-up between the integrating firms. The elimination of bilateral contracting externalities has two effects: it raises investment for any given prices, and it raises prices for any given investment. The net effect on welfare of this effect alone is ambiguous. The elimination of hold-up unambiguously raises investment in our model, and this effect alone benefits consumers. Our model captures all these effects.

Arm data is for the period April 1, 2018 to March 1, 2019 and is from "SoftBank-backed chip maker ARM spends £655m on R&D", available at https://finance.yahoo.com/news/arm-holdings-2019-accounts-softbank-r-and-d-120402342.html, site accessed March 25, 2021.

We showed that after accounting for these effects, consumers benefit from the merger if the differentiation between downstream products is sufficiently strong or sufficiently weak. When differentiation is strong, bilateral contracting externalties are small, but the investment benefits of eliminating hold-up are positive. If differentiation is sufficiently weak, bilateral contracting externalties are strong, so strong that a potential entrant stays out of the market pre-merger. In this case, the merger raises investment by eliminating hold-up, and it may encourage entry and additional investment.

For intermediate degrees of product differentiation, the effect of the merger on consumers depends on the relative importance of investment for generating value. Under a variant of the model in which investments shift a Levitan-Shubik demand function and investment costs are quadratic, we showed that the merger benefits consumers if the marginal cost of investment is sufficiently low, as revealed by large R&D expenditures relative to sales revenues.

We applied our model to examine the vertical aspects of the NVIDIA-Arm merger in the market for computer chips sold to datacenters. Arm is an upstream licensor of chip technology to NVIDIA and a competitor who use the license to design chips and servers used in datacenters. In the absence of investment considerations, a standard model of bilateral contracting with secret contracts would predict that the merged firm would raise royalties to the unintegrated downstream firm, leading to higher prices and lower consumer welfare. However, when we incorporate upstream and downstream investment into the model in a straightforward way, where investment shifts a LS demand at a quadratic cost, we showed that the model predicts that the merger would increase investment and benefit consumers. Although the model admits the possibility that the price effects of eliminating bilateral contracting externalities could harm consumers, the investment-to-sales ratio in the high performance chip industry falls in a range where the model predicts that the merger would likely be procompetitive.

Our model relies on assumptions that are worthy of further exploration. First, we assume that the upstream supplier cannot commit to refrain from bilateral negotiations over contract terms with each downstream firm, an assumption that is known to be important in the economics of vertical contracting. For example, suppose the supplier could commit to charge the same per unit price but still negotiated fixed fees bilaterally. One might conjecture that the ability to make such a commitment would lead to the fully integrated outcome prior to the merger, but that is not likely when both price and investment matter because wholesale prices cannot control both, and it might not be likely even in the absence of investment considerations because the supplier may want to distort the wholesale price to favorably affect disagreement profits in negotiations with downstream firms. It is unclear without additional work what distortions would arise or how they would interact with investment and price effects.

Second, we assume that firms can write perfect nonlinear contracts. An alternative assumption worth exploring is linear contracting. It might be thought that such contracting would lead to pre-merger double marginalization and make vertical mergers more likely to be procompetitive. However, this is not obvious if downstream firms have bargaining power and negotiate prices bilaterally, because bilateral bargaining over linear prices generates contracting externalities that can lower pre-merger wholesale prices and joint profits in a way similar to the private bilateral nonlinear contracts we analyze.[30]

Third, our model considers the effects of the merger the entry decision of an independent downstream firm, but we do not consider the potential for the merged firm to introduce a new product. If the merged firm would benefit more from such entry than a third party, it seems possible that the benefits of a merger could be larger than what we have described. The role of commitments, contractual form, and lumpy investment (e.g., entry) by the merged firm in determining the effects of vertical mergers when investment and price are both important strategies are topics for further research.

[30]See the references in footnote 2.

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